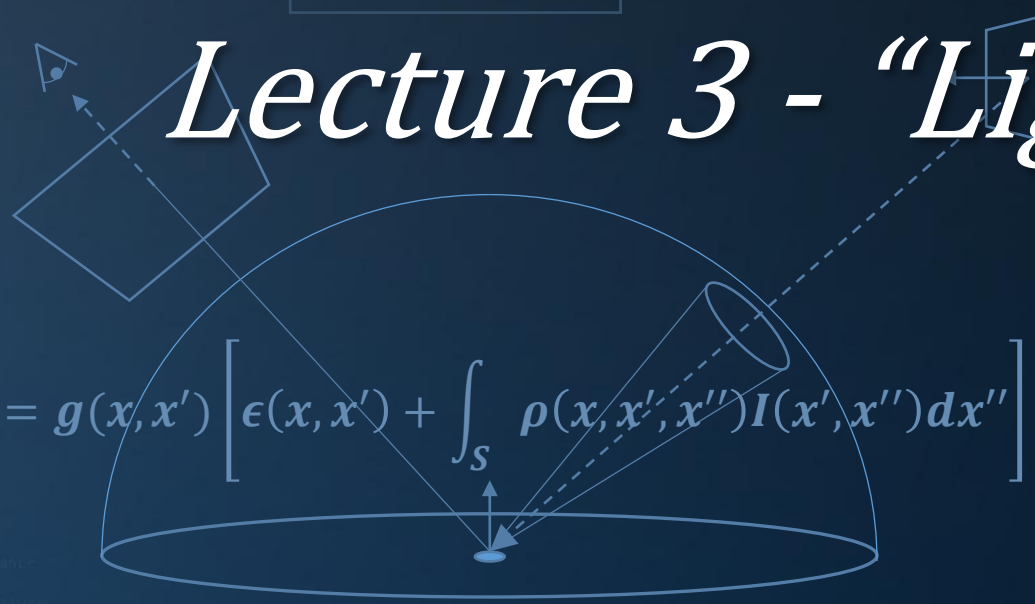
INFOMAGR – Advanced Graphics

Jacco Bikker - November 2022 - February 2023

Lecture 3 - "Light Transport"



$$I(x,x') = g(x,x')\left[\epsilon(x,x') + \int_S \rho(x,x',x'')I(x',x'')dx''\right]$$

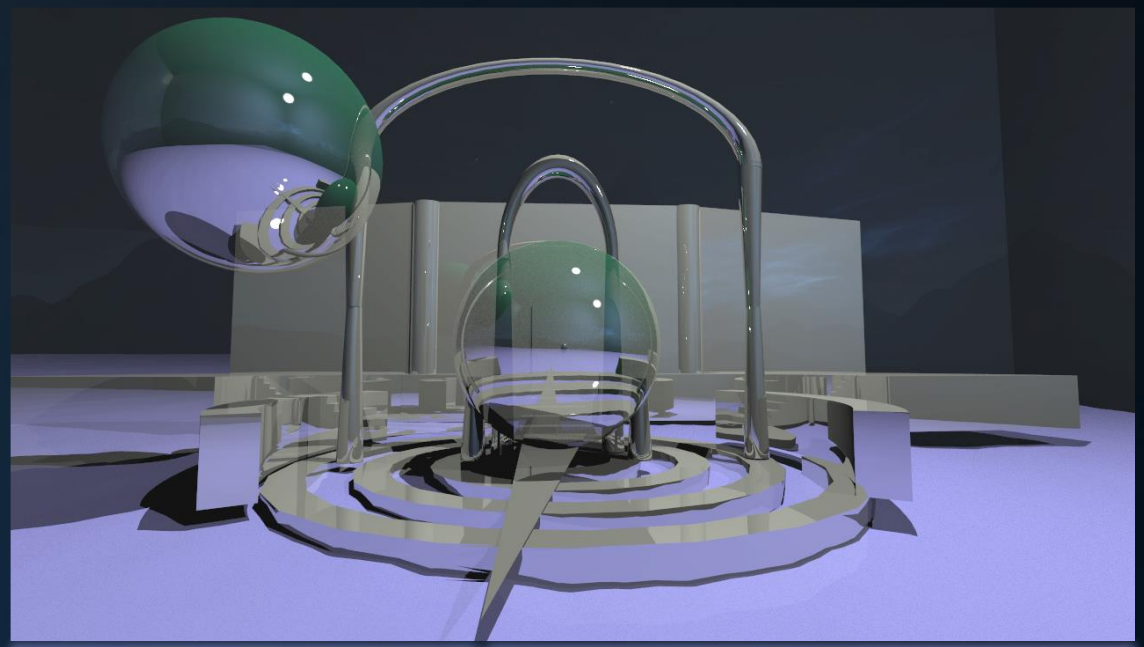
Welcome!

Today’s Agenda:

- Introduction
- The Rendering Equation
- Light Transport

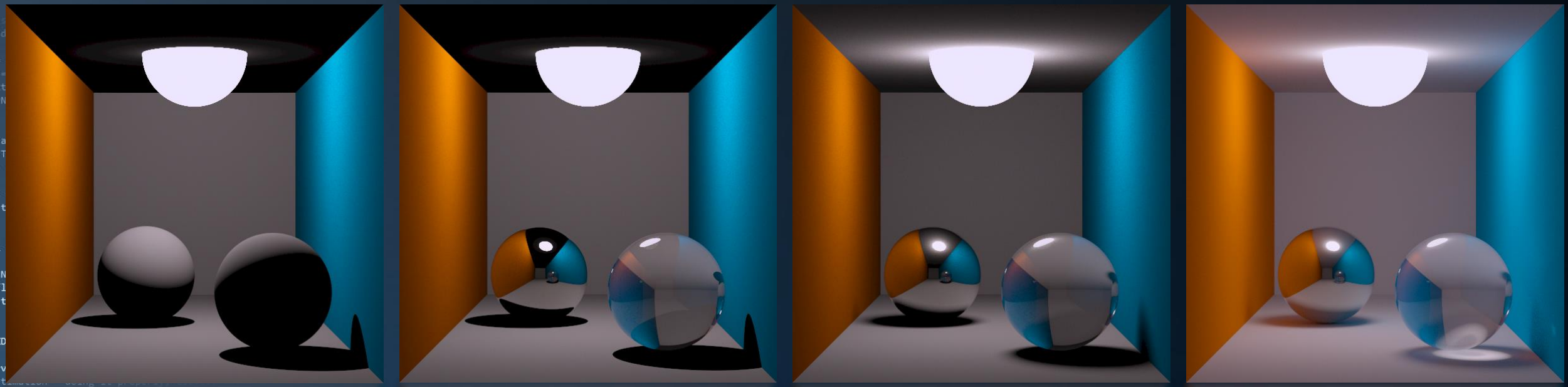
Introduction

Whitted



Introduction

Whitted

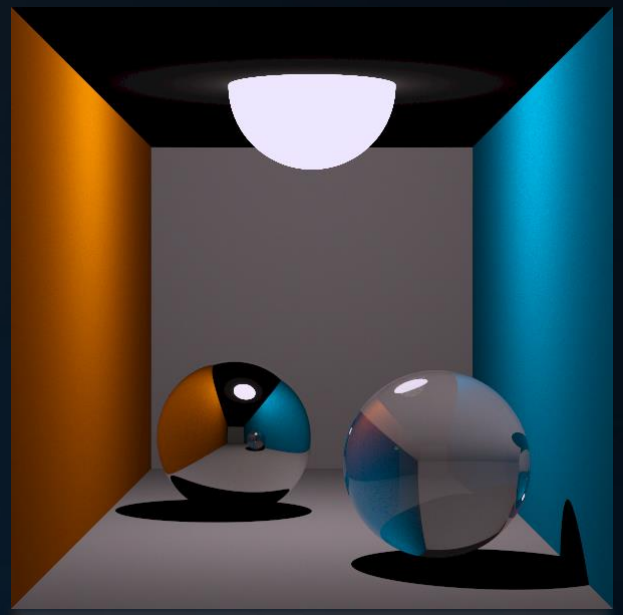


Introduction

Whitted

Missing:

- Area lights
- Glossy reflections
- Caustics
- Diffuse interreflections
- Diffraction
- Polarization
- Phosphorescence
- Temporal effects
- Motion blur
- Depth of field
- Anti-aliasing



Introduction

Anti-aliasing

Adding anti-aliasing to a Whitted-style ray tracer:

Send multiple primary rays through each pixel, and average their result.

Problem:

- How do we aim those rays?
- What if all rays return the same color?

Introduction

Anti-aliasing – Sampling Patterns

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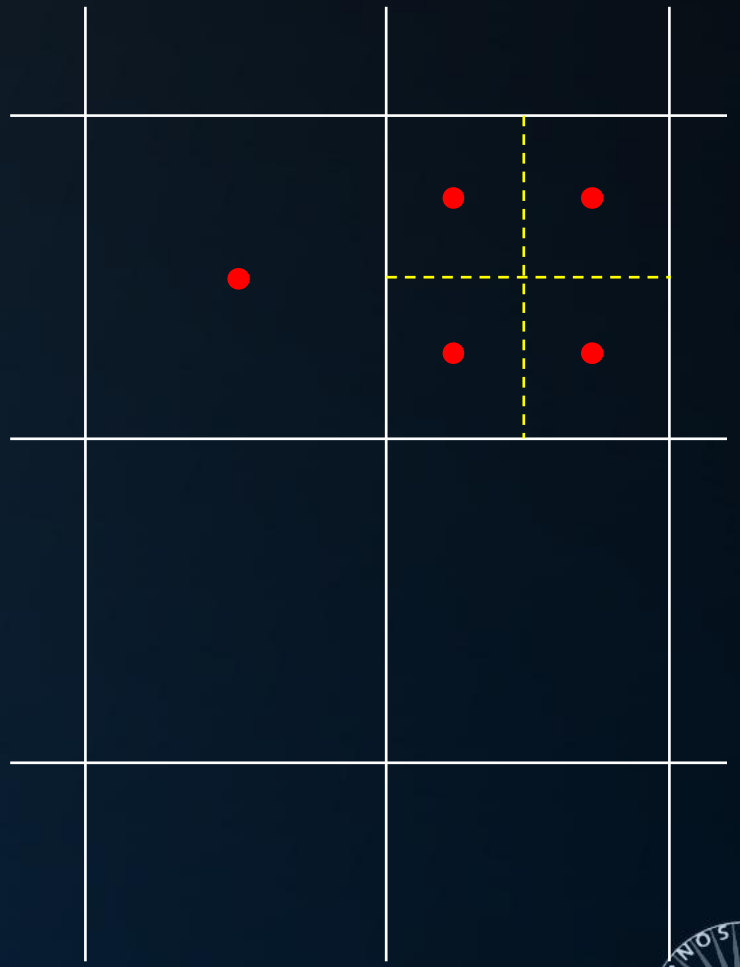
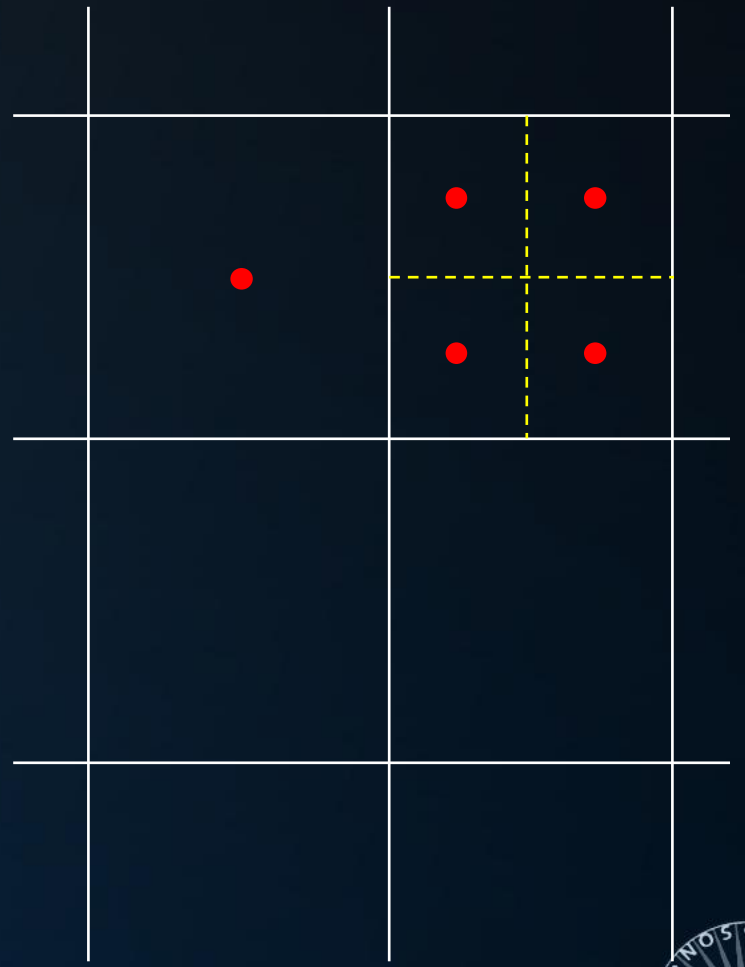


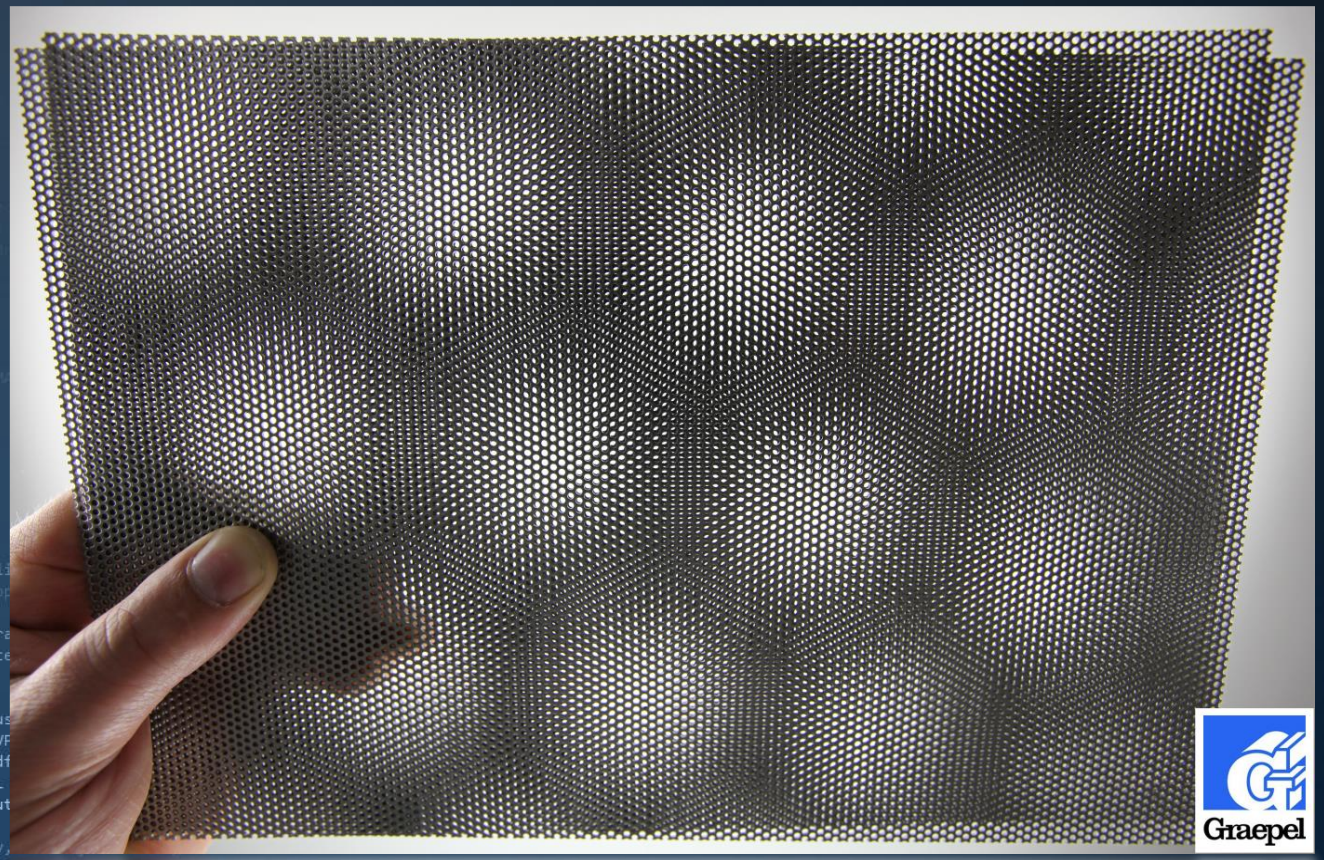
Introduction

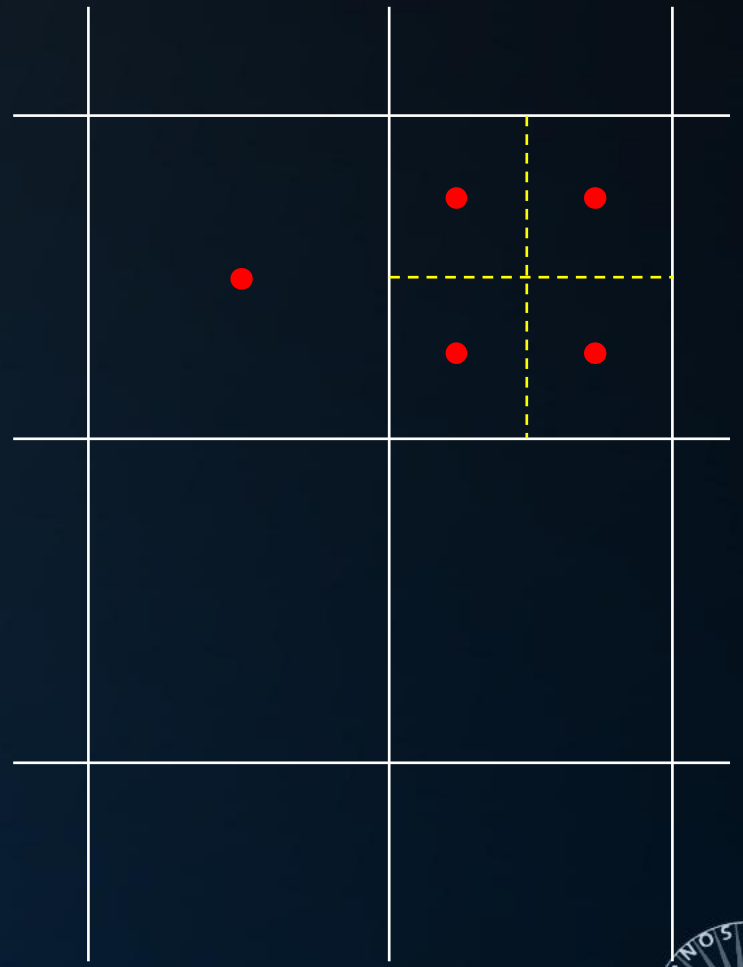
Anti-aliasing – Sampling Patterns



Introduction

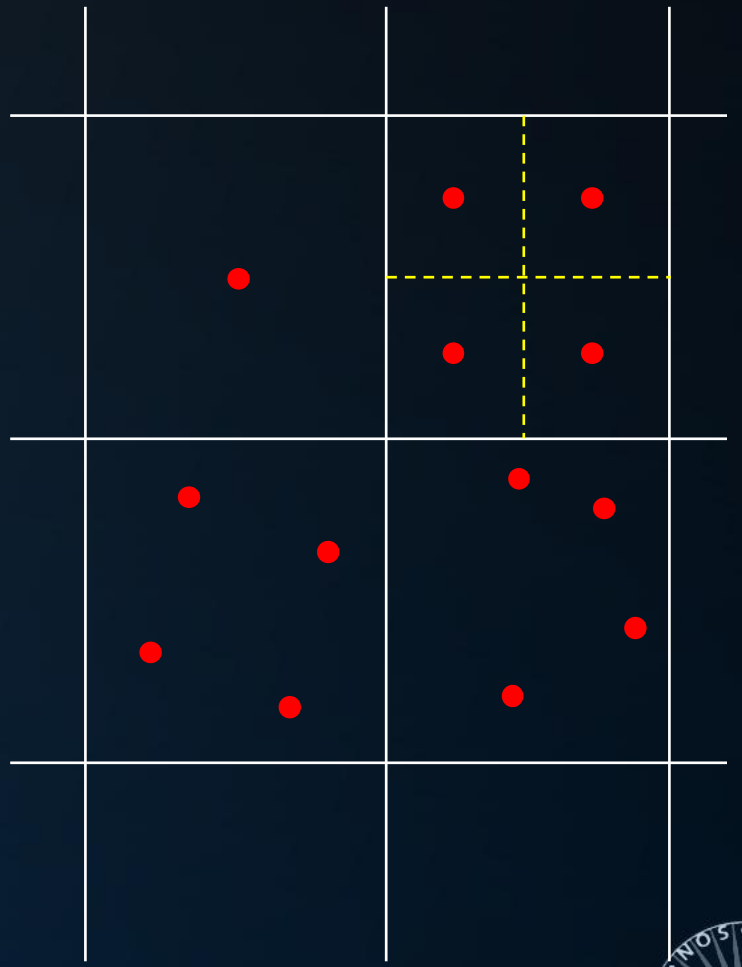
Anti-aliasing – Sampling Patterns





Introduction

Anti-aliasing – Sampling Patterns



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Anti-aliasing – Sampling Patterns

Adding anti-aliasing to a Whitted-style ray tracer:

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More info: https://mynameismjp.wordpress.com/2012/10/24/msaa-overview

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- Phosphorescence
- Temporal effects
- Motion blur
- Depth of field
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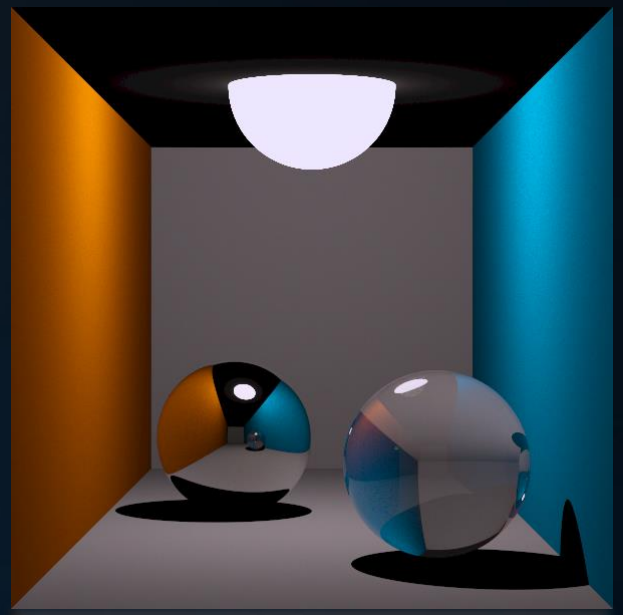
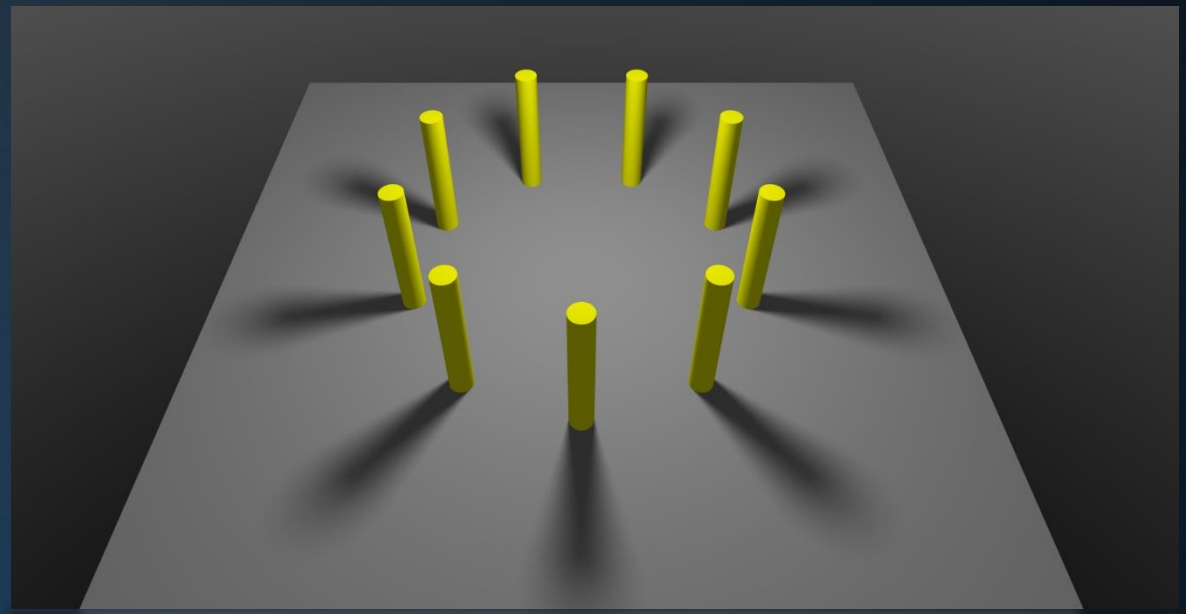
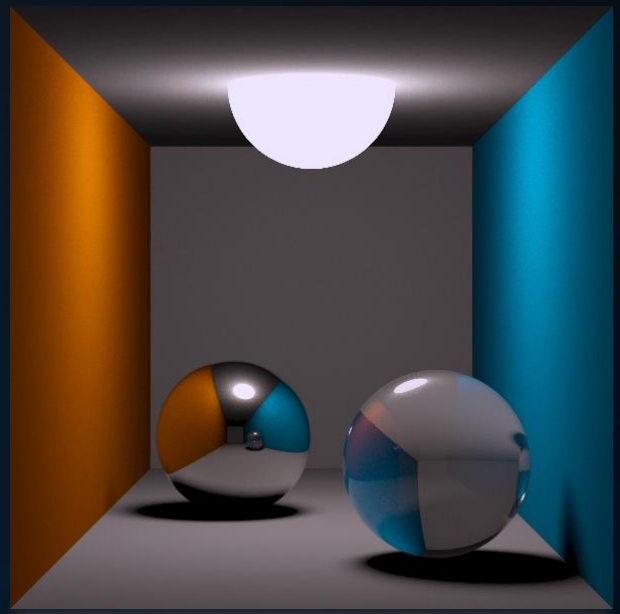


Introduction

Distribution Ray Tracing*



Soft shadows

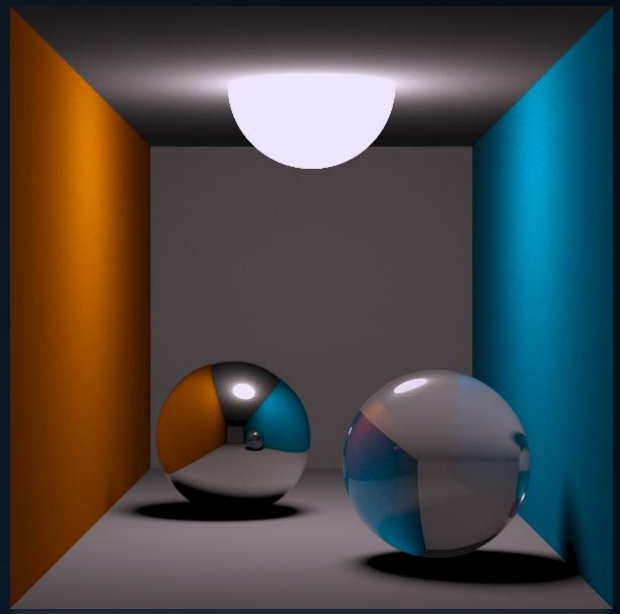


*: Distributed Ray Tracing, Cook et al., 1984

Introduction

Distribution Ray Tracing*

Glossy reflections



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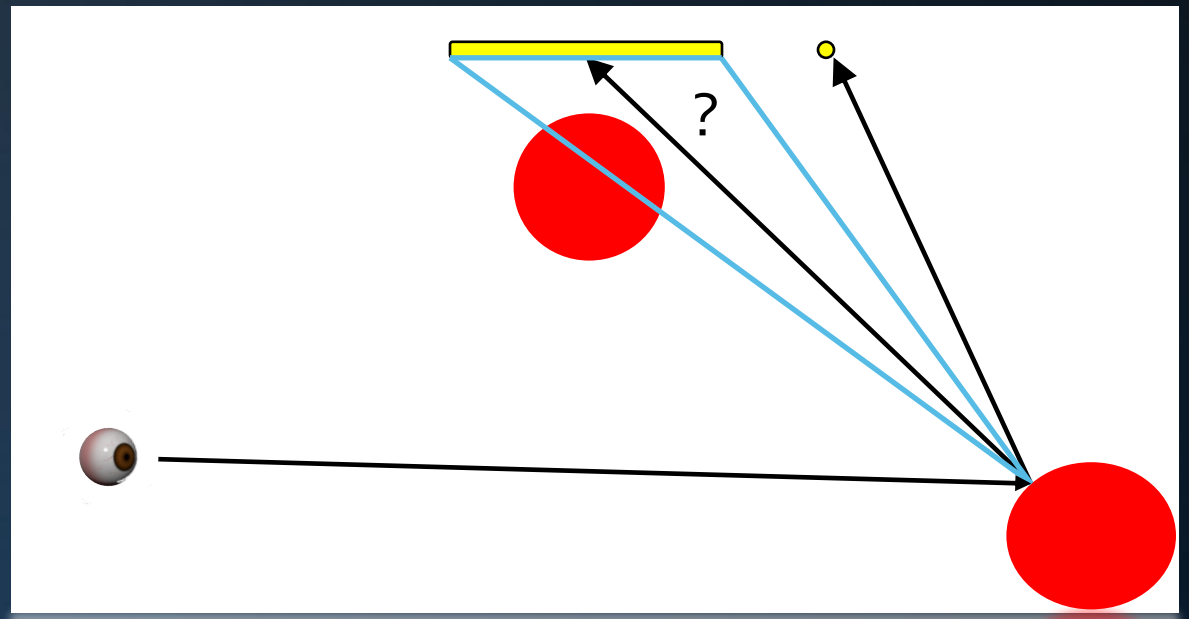

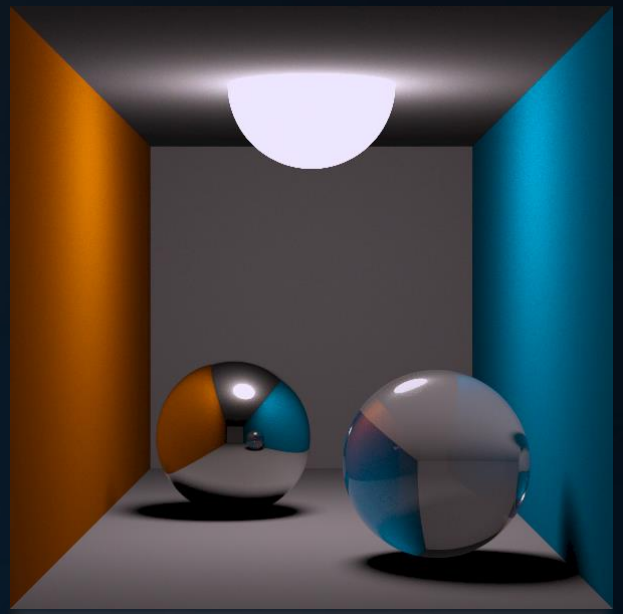

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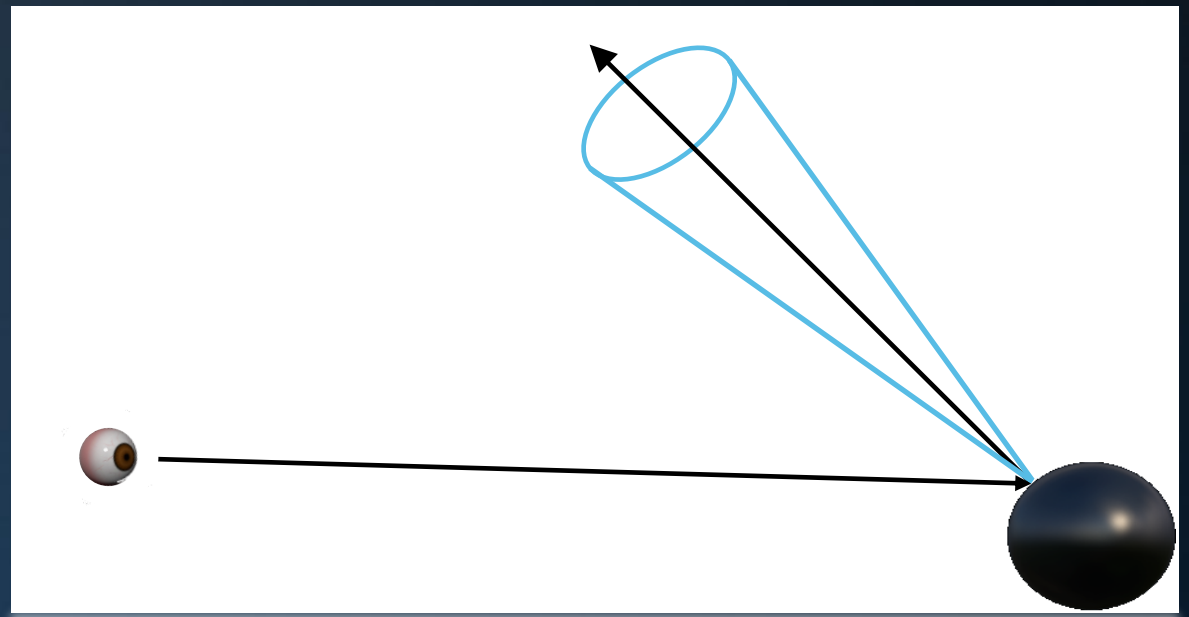
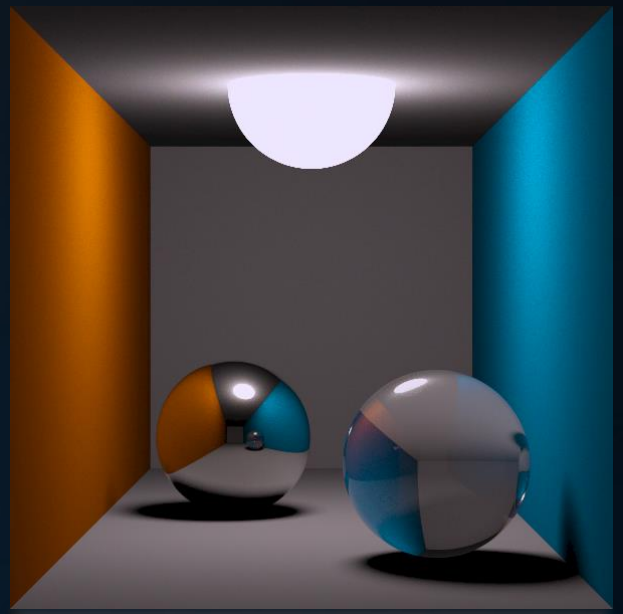
Distribution Ray Tracing*





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Introduction

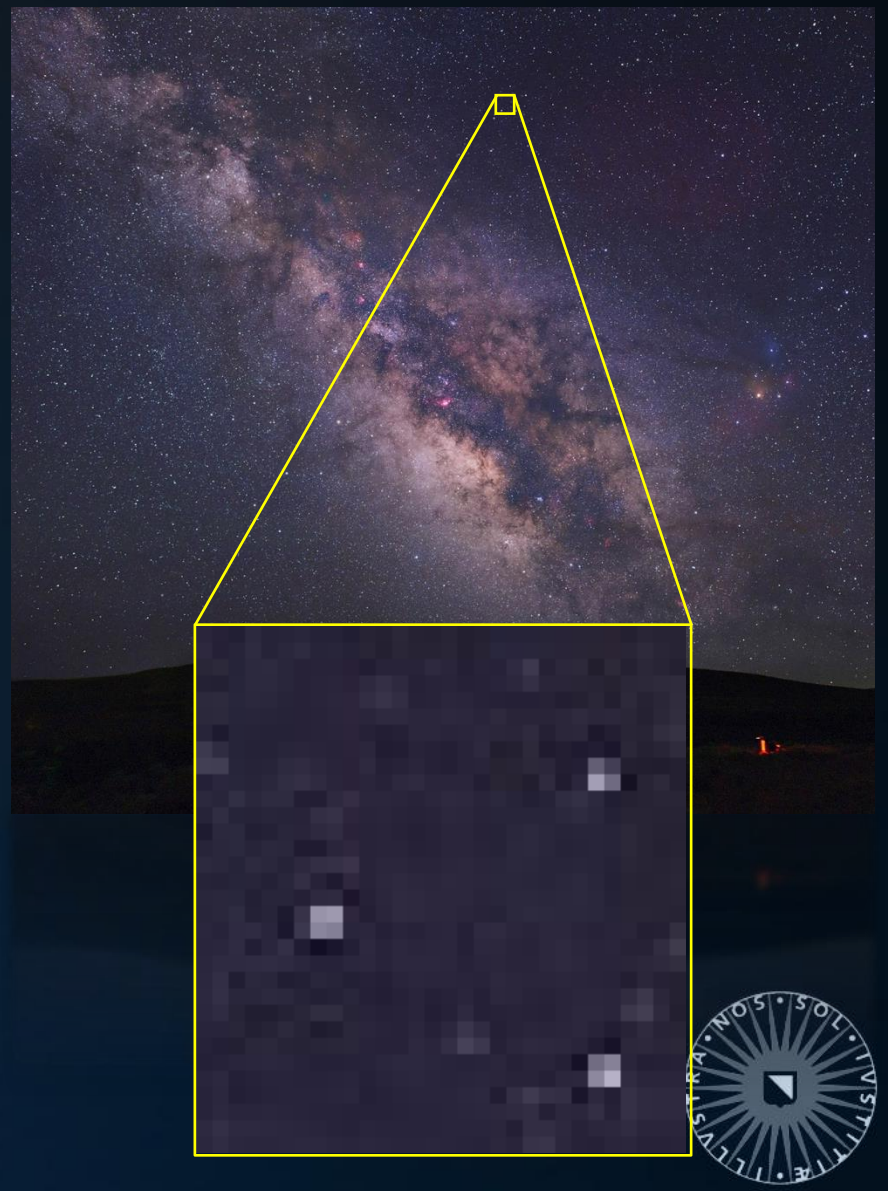
Distribution Ray Tracing

Whitted-style ray tracing is a *point sampling* algorithm:

- We may miss small features
- *We cannot sample areas*

Area sampling:

- Anti-aliasing: one pixel
- Soft shadows: one area light source
- Glossy reflection: directions in a cone
- Diffuse reflection: directions on the hemisphere



Introduction

Area Lights

Visibility of an area light source:

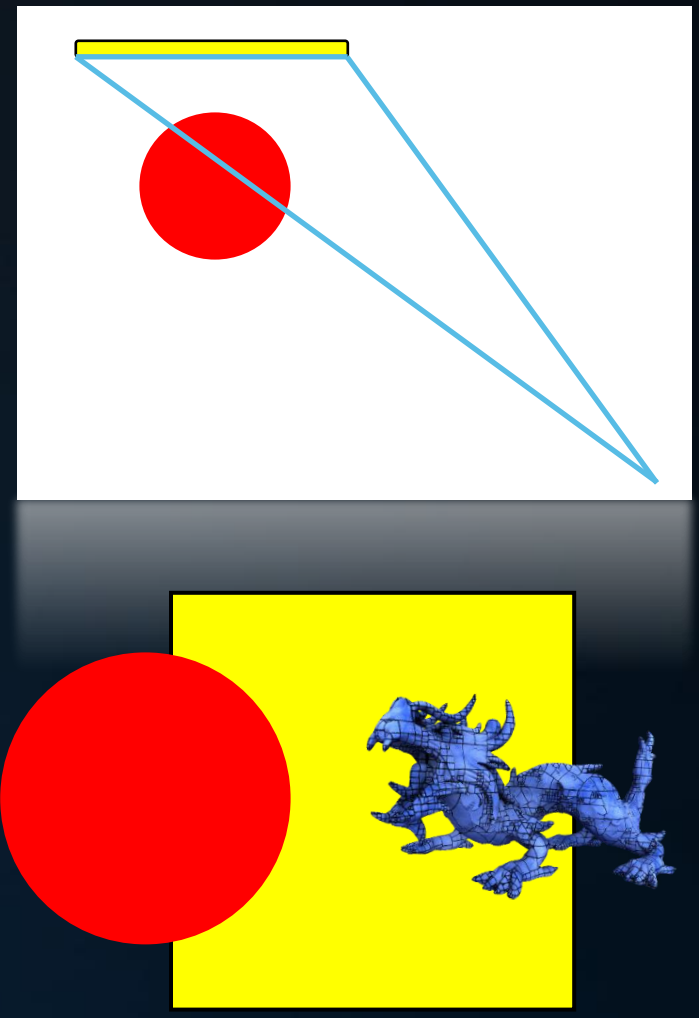
$$V_A = \int_A V(x, \omega_i)\, d\omega_i$$

Analytical solution case 1:

$$V_A = A_{light} - A_{light \cap sphere}$$

Analytical solution case 2:

$$V_A = \ ?$$



Introduction

Approximating Integrals

An integral can be approximated as a Riemann sum:

$$V_A = \int_A^B f(x)\,dx \approx \sum_{i=1}^{N} f(t_i)\,\Delta_i\,, \text{where} \sum_{i=1}^{N} \Delta_i = B - A$$

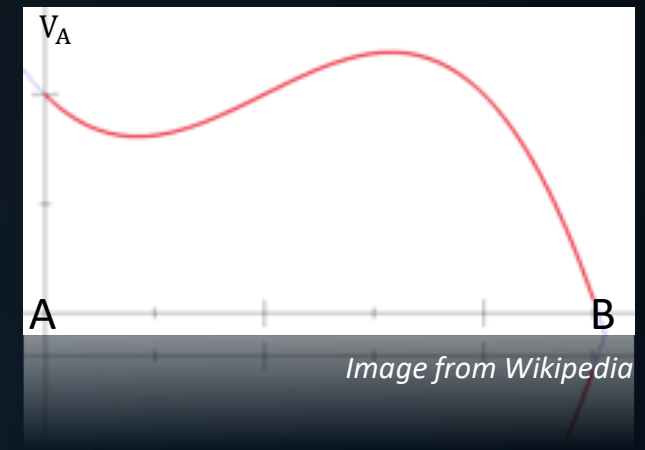
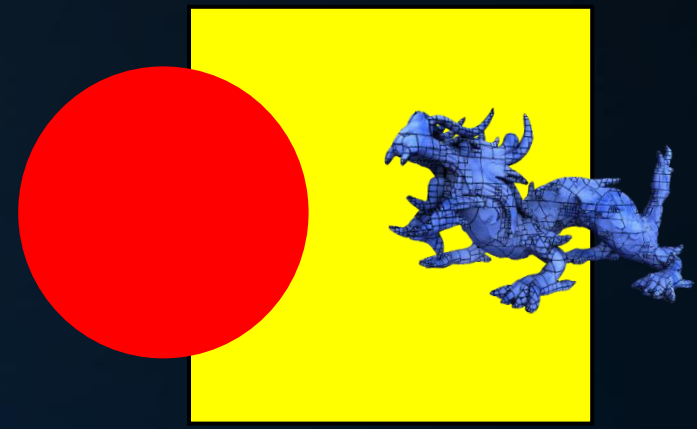


Image from Wikipedia

Note that the intervals do not need to be uniform, as long as we sample the full interval. If the intervals are uniform, then

$$\sum_{i=1}^{N} f(t_i)\,\Delta_i = \Delta_i \sum_{i=1}^{N} f(t_i) = \frac{B-A}{N} \sum_{i=1}^{N} f(t_i)\,.$$



Regardless of uniformity, restrictions apply to N when sampling multi-dimensional functions (ideally, $N = M^d, M \in \mathbb{N}$). Also note that aliasing may occur if the intervals are uniform.

Introduction

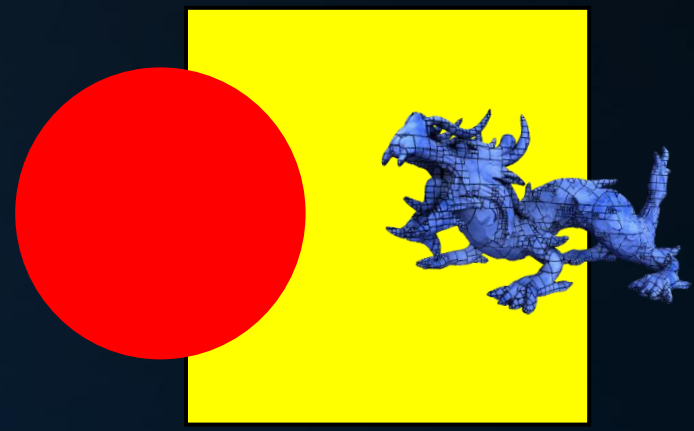
Monte Carlo Integration

Alternatively, we can approximate an integral by taking random samples:

$$V_A = \int_A^B f(x)\,dx \approx \frac{B-A}{N}\sum_{i=1}^{N} f(X_i)$$

Here, $X_1..X_N \in [A,B]$.
As N approaches infinity, V_A approaches the *expected value* of f(X).

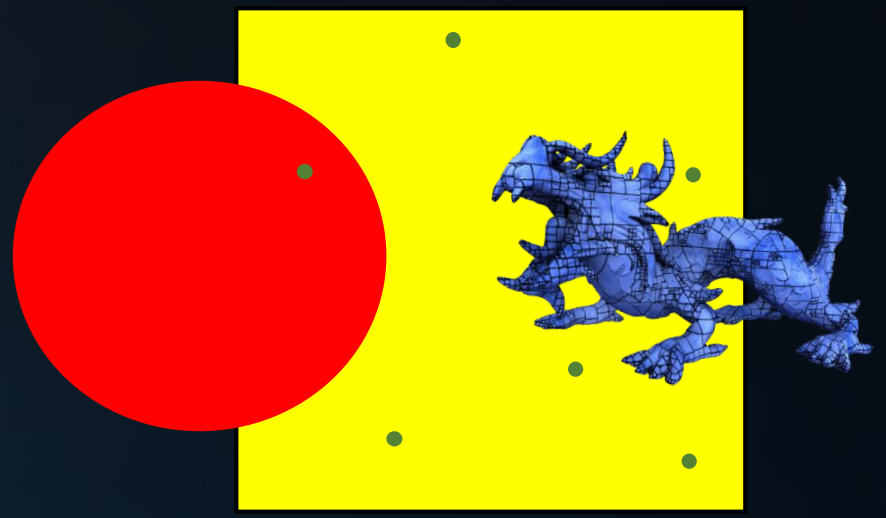
Unlike in Riemann sums, we can use arbitrary N for Monte Carlo integration, regardless of dimension.



Introduction

Monte Carlo Integration of Area Light Visibility

To estimate the visibility of an area light source, we take N random point samples.



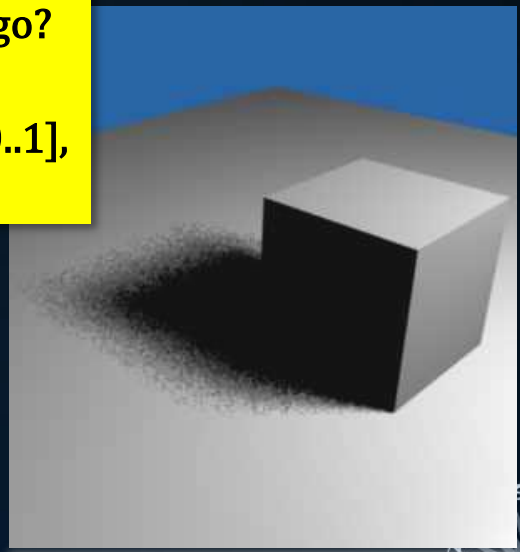
In this case, 5 out of 6 samples are unoccluded:

$$V \approx \frac{1}{6}(1+1+1+0+1+1) = \frac{5}{6}$$

Properly formulated using a MC integrator:

$$V = \int_{S^2} V(p)\,dp \approx \frac{1}{N}\sum_{i=1}^{N} V(P)$$

Q: Where did the $\frac{B-A}{N}$ go?
A: the domain of the visibility function is [0..1], so $B - A = 1$.



With a small number of samples, the variance in the estimate shows up as noise in the image.

Introduction

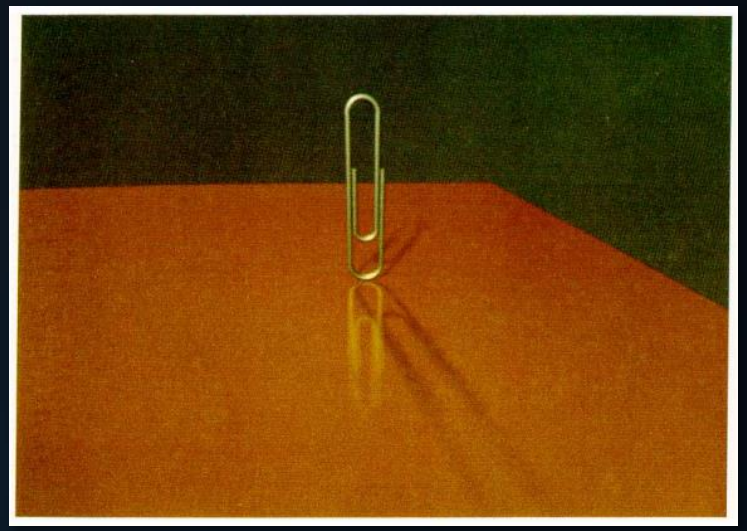
Distribution Ray Tracing

Key concept of distribution ray tracing:

We estimate integrals using Monte Carlo integration.

Integrals in rendering:

- Area of a pixel
- Lens area (aperture)
- Frame time
- Light source area
- Cones for glossy reflections
- Wavelengths
- Many lights
- …



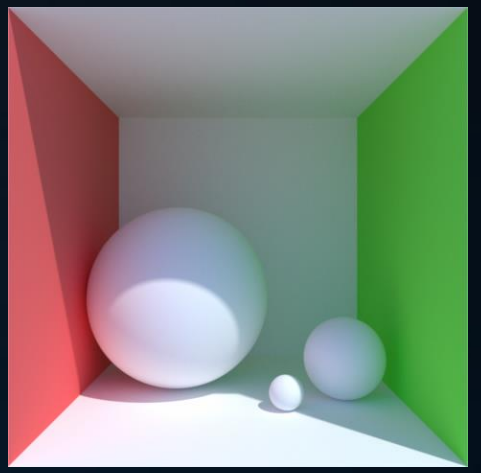
Today’s Agenda:

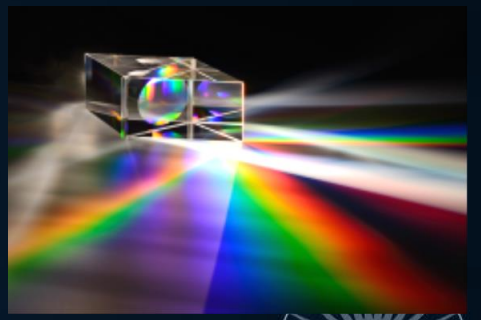
- Introduction
- The Rendering Equation
- Light Transport

Rendering Equation

Whitted, Cook & Beyond

Missing in Whitted:	Cook:
▪ Area lights	✓ Area lights
▪ Glossy reflections	✓ Glossy reflections
▪ Caustics	✗ Caustics
▪ Diffuse interreflections	✗ Diffuse interreflections
▪ Diffraction	✗ Diffraction
▪ Polarization	✗ Polarization
▪ Phosphorescence	✗ Phosphorescence
▪ Temporal effects	✗ Temporal effects
▪ Motion blur	✓ Motion blur
▪ Depth of field	✓ Depth of field
▪ Anti-aliasing	✓ Anti-aliasing





Rendering Equation

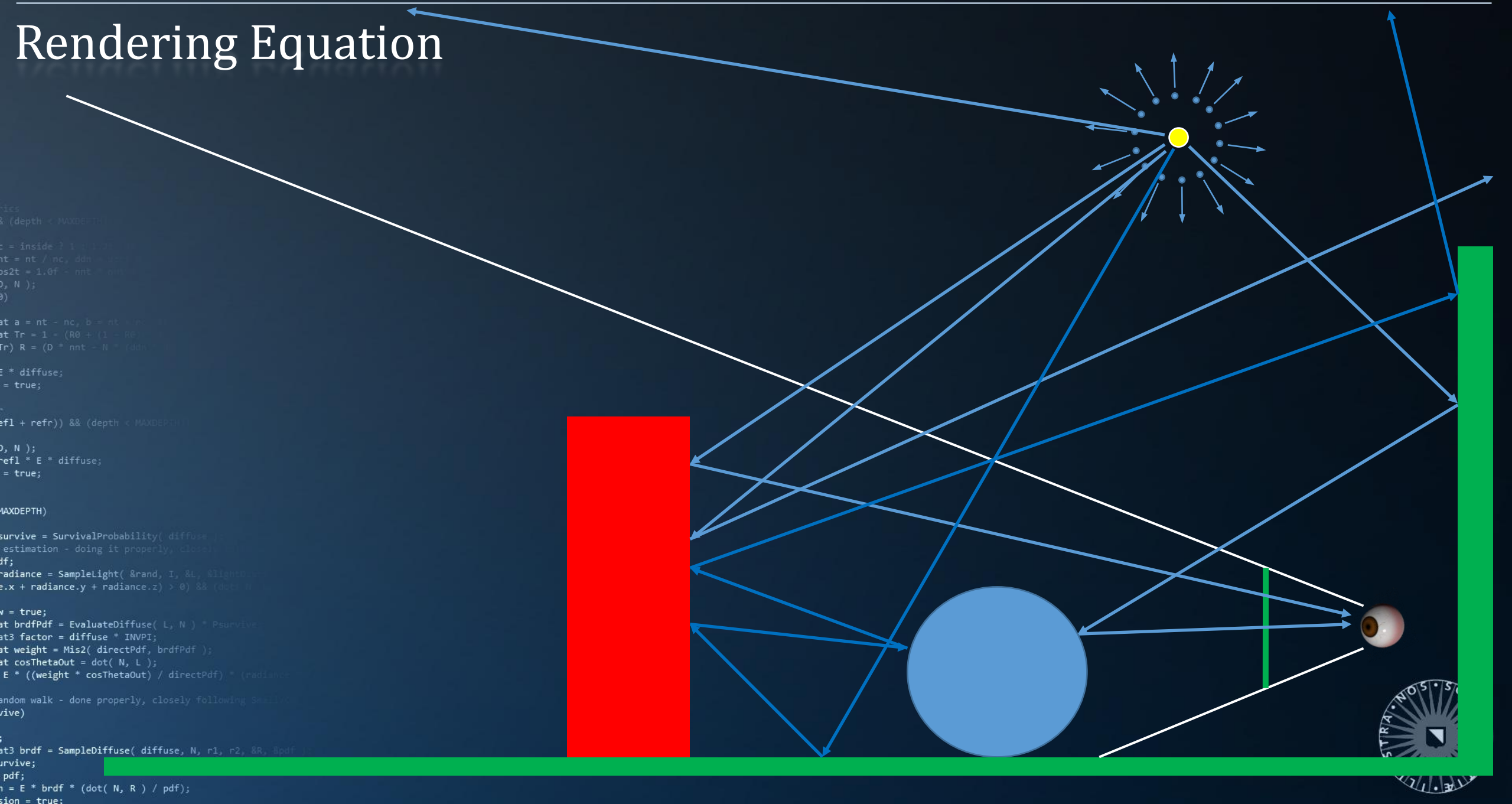
God's Algorithm

1 room
1 bulb
100 watts
10^{20} photons per second

Photon behavior:

- Travel in straight lines
- Get absorbed, or change direction:
  - Bounce (random / deterministic)
  - Get transmitted
- Leave into the void
- Get detected

Rendering Equation



Rendering Equation

God's Algorithm - Mathematically

A photon may arrive at a sensor after travelling in a straight line from a light source to the sensor:

$$L(s \leftarrow x) = L_E(s \leftarrow x)$$

Or, it may be reflected by a surface towards the sensor:

$$L(s \leftarrow x) = \int_A f_r(s \leftarrow x \leftarrow x')\, L(x \leftarrow x')\, G(x \leftrightarrow x')\, dA(x')$$

Those are the options.
Adding direct and indirect illumination together:

$$L(s \leftarrow x) = L_E(s \leftarrow x) + \int_A f_r(s \leftarrow x \leftarrow x')\, L(x \leftarrow x')\, G(x \leftrightarrow x')\, dA(x')$$

Rendering Equation

God's Algorithm - Mathematically

$$L(s \leftarrow x) = L_E(s \leftarrow x) + \int_A f_r(s \leftarrow x \leftarrow x')\, L(x \leftarrow x')\, G(x \leftrightarrow x')\, dA(x')$$

Emission

Hemisphere

Reflection

Indirect

Geometry factor

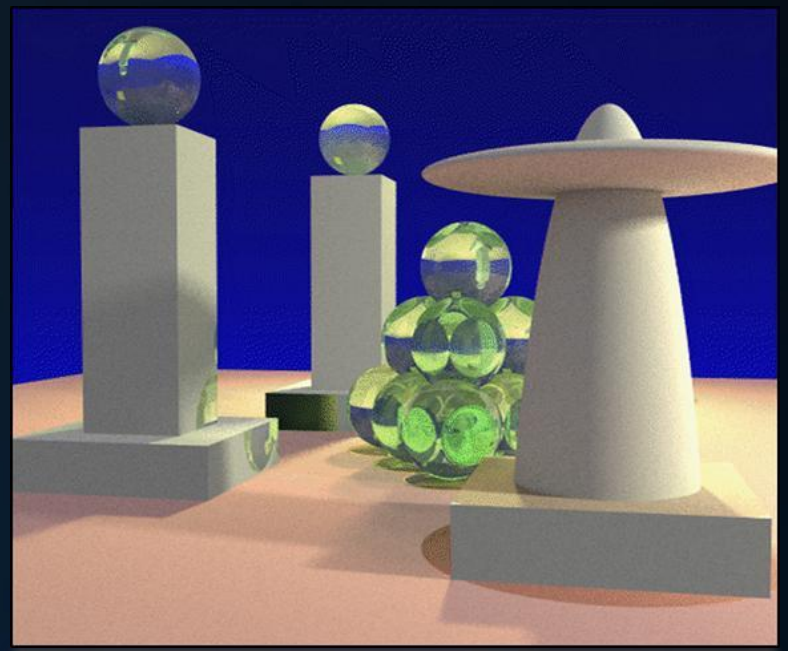
Rendering Equation

$$L(s \leftarrow x) = L_E(s \leftarrow x) + \int_A f_r(s \leftarrow x \leftarrow x')\, L(x \leftarrow x')\, G(x \leftrightarrow x')\, dA(x')$$

The Rendering Equation*:

- Light transport from lights to sensor
- Recursive
- Physically based

The equation allows us to determine to which extend rendering algorithms approximate real-world light transport.



*: The Rendering Equation, Kajiya, 1986

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END of “Light Transport”

next lecture: “Path Tracing”